

Entropy – A New Perspective On Study Of Randomness Of universe



DISSERTATION

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE AWARD OF THE DEGREE OF
BACHELOR OF SCIENCE

DEPARTMENT OF PHYSICS

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ROLL NO.- 221

SESSION (2022-25)

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DECLARATION

I hereby declare that the work which is presented in this Dissertation Entropy – A New Perspective In Study Of Randomness Of Universe in partial fulfillment of the requirement for the award of the Degree of Bachelor of Science in PHYSICS (DSE-04), Patna Science College, Patna University, Patna, has been carried out during the period from January 2025 to May 2025 under the supervision of Prof. Mukesh Kumar of the Department of PHYSICS.

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Entropy—A New Perspective on the Study of Randomness of the Universe

Abstract

Entropy, a fundamental concept linking thermodynamics, information theory, and cosmology, quantifies the randomness or disorder inherent in physical systems. This thesis investigates entropy across three domains: thermodynamics (the science of heat and work), information theory (the science of data and information), and cosmology (the study of the universe's origin and evolution). By analyzing the conceptual and mathematical underpinnings of entropy in each field, we reveal deep connections: thermodynamic entropy measures accessible microstates of a system (Clausius–Boltzmann framework), Shannon entropy quantifies uncertainty in information sources, and cosmological entropy governs the universe's large-scale state and

ultimate fate. We develop a unified perspective showing that in all cases entropy manifests as a logarithmic measure of multiplicity or uncertainty. We present mathematical formalisms (e.g. $S = k_B \ln \Omega$, Shannon's $H = -\sum p_i \log_2 p_i$, Bekenstein-Hawking black-hole entropy $S = k_B A / 4 \ell_P^2$) and interpret them physically. We include illustrative figures: distributions whose shapes relate to entropy, and cosmic microwave background maps reflecting early-universe "randomness". Our results underscore that the universe's perceived randomness is grounded in statistical laws of entropy, with thermodynamic irreversibility and information uncertainty as two sides of the same coin. This thesis also reviews entropy's role in the arrow of time and heat death, and discusses open questions like the black-hole information paradox. Finally, we suggest that viewing entropy as a unifying measure of randomness offers novel insights into the universe's behavior on all scales.

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List of Symbols

- S – Thermodynamic entropy (J/K), a state function measuring disorder/unusable energy.
- k_B – Boltzmann constant ($\approx 1.38 \times 10^{-23}$ J/K) relates energy to temperature.
- Ω – Number of microstates corresponding to a macrostate in statistical mechanics.
- $H(X)$ – Shannon entropy of a random variable X (in bits or nats).
- P – Probability of state i in an ensemble or distribution.
- A – Black hole horizon area (in m^2).
- ℓ_P – Planck length ($\approx 1.62 \times 10^{-35}$ m).
- c – Speed of light
- $\hbar$ – Reduced Planck constant
- G – Gravitational constant.
- Λ – Cosmological constant (dark energy density parameter).
- T – Absolute temperature (K).

- Q – Heat transfer (J).
- ΔS – Change in entropy (J/K).
- X – A generic random variable with distribution $\{p_i\}$

1. Introduction

Entropy is a cornerstone of modern physics that measures the amount of **randomness**, **uncertainty**, or **disorder** in a system. In classical thermodynamics, entropy determines the direction of spontaneous processes: isolated systems evolve toward states of higher entropy, reflecting an inexorable increase of disorder. Shannon later generalized the idea to communication theory, defining entropy as the average uncertainty of information-bearing signals. In cosmology, the concept of entropy takes on a grand scale: it underlies the arrow of time and the fate of the universe (e.g. heat death scenario). This thesis explores entropy *across* these domains, treating entropy not just as a physical quantity in one context but as a **unifying perspective on randomness** throughout nature.

We begin by examining the classical definition of entropy, $S = K_B \ln \Omega$. where Ω is the number of possible

microstates of a system. This logarithmic measure encapsulates how microscopic arrangements manifest as macroscopic irreversibility. We then introduce Shannon entropy,

$$H(X) = -\sum P_i \log_2 P_i$$

which quantifies the expected information content (uncertainty) of a random variable. Remarkably, the mathematical forms are analogous: both measure “effective number” of states (microstates or outcomes). Our analysis will show that by focusing on this common structure, one gains new insights into how the universe’s overall randomness is shaped by thermodynamic laws and information constraints alike.

We also integrate cosmology by considering entropy on cosmic scales. The universe began in an extraordinarily **low-entropy** state (highly uniform Big Bang), and its entropy has been increasing ever since. The thermodynamic arrow of time is thus intimately tied to cosmic evolution. For example, as the universe expands

and forms structure (galaxies, stars), local decreases in entropy (clumping under gravity) occur only by dumping even more entropy into radiation and black holes. The largest entropy reservoirs in the cosmos are supermassive black holes: their entropy scales with horizon area, $S_{\text{BH}} = K_{\text{B}} A / 4l_{\text{P}}^2$ which dwarfs that of ordinary matter or radiation.

This thesis follows a traditional academic structure. In **Chapter 2**, we review the literature on entropy in thermodynamics, information theory, and cosmology, highlighting key definitions, equations, and interpretations. **Chapter 3** explains our methodology, primarily conceptual and mathematical analysis, in unifying the domains. **Chapter 4** presents results and analysis: we derive and compare entropy formulas, illustrate entropy with figures (including probability distributions and cosmic background maps), and compile numerical estimates such as the cosmic entropy budget. **Chapter 5** discusses the implications of these findings, such as the universality of statistical laws, and

unresolved issues like the black hole information paradox. Finally, **Chapter 6** concludes by summarizing the *new perspective* gained: entropy as a single conceptual thread linking microphysics, information, and the fate of the universe.

2. Literature Review

2.1 Thermodynamic Entropy

In classical thermodynamics, entropy S was introduced by Rudolf Clausius in the 19th century to formalize the second law of thermodynamics. An isolated system's entropy **never decreases**; indeed, any irreversible process increases total entropy. Formally, for a reversible heat transfer dQ at temperature T , the change in entropy is defined as $dS = dQ_{\text{rev}}/T$

Boltzmann later provided a microscopic interpretation: $S = k_B \ln \Omega$, where Ω is the number of accessible microstates compatible with the macrostate. This relationship illuminates entropy as a measure of disorder or randomness: more microstates (greater Ω) correspond to higher entropy.

Entropy's role extends to energy availability: only part of a system's internal energy can do work, the rest is

“unavailable” and contributes to entropy. A classic example is heat flowing from hot to cold; entropy increases as energy disperses. Textbooks and reviews consistently emphasize these points: Clausius and Gibbs established the foundations (see Gibbs’ formula $S = -k_B \sum p_i \ln p_i$ in equilibrium), while modern pedagogical sources illustrate entropy with mixing of gases or melting ice. One practical exposition notes: *“the entropy of a system can be shown to be a measure of its disorder and of the unavailability of energy to do work”*.

Entropy also provides an **arrow of time**: since isolated systems tend toward maximum entropy, this defines a direction (past with low SS, future with high SS). As Britannica explains, *“the implication is that the universe must ultimately suffer a ‘heat death’ as its entropy progressively increases toward a maximum value and all parts come into thermal equilibrium”*. Here entropy is equated with disorder: at thermal equilibrium (maximum entropy), no macroscopic processes can occur.

In summary, thermodynamic entropy is a physical quantity with both statistical and phenomenological meanings. It enforces irreversible behavior in macroscopic systems and is quantitatively given by $S = k_B \ln \Omega$, a logarithmic measure of microscopic configurations. This classical framework will serve as a baseline for comparison with information and cosmology.

2.2 Information-Theoretic Entropy

Claude Shannon introduced a mathematical analog of entropy in 1948 to address communication theory. Shannon entropy $H(X)$ measures the average unpredictability of a random source X . For a discrete variable with probability distribution $\{p_i\}$,

$$H(X) = -\sum P_i \log_2 P_i$$

(based on any log base; base-2 gives units of bits).

Shannon's formula emerged from axiomatic

requirements (continuity, maximality for uniform distribution, and additivity) and was proven to quantify information content. Essentially, $H(X)$ is the *expected* number of bits needed to encode outcomes of X .

Conceptually, Shannon entropy captures **uncertainty** and **randomness** in data. A fair coin toss (with $p_1=p_2=0.5$) has $H=1$ bit, the maximal entropy for two outcomes. If the distribution is skewed (e.g. $p_1=0.99$, $p_2=0.01$), H is much lower. Shannon's foundational theorems link this to compression: one cannot encode messages with fewer bits on average than the source entropy.

Interestingly, Shannon entropy is formally analogous to thermodynamic entropy (Gibbs entropy). As Wikipedia notes, when probabilities p_i represent normalized Boltzmann weights (i.e. $p_i \propto e^{-\epsilon_i/k_B T}$), Shannon's H becomes Gibbs' entropy S/k_B . Both count effective states with a logarithm. Thus, information entropy can be viewed as a generalization of disorder to any

probabilistic context. The literature emphasizes this bridge: “Entropy in information theory is directly analogous to the entropy in statistical thermodynamics”.

Modern articles and blogs underscore Shannon’s innovation: numberanalytics.com explains that Shannon extended thermodynamic uncertainty to data, defining $H(X)$ as above. They also contrast the domains: thermodynamic entropy deals with energy dispersal and molecular disorder, whereas Shannon entropy measures uncertainty in message outcomes. Despite these contextual differences, both use logarithmic measures. We will build on this analogy: in subsequent chapters, statistical ensembles in physics and data sources in communication will be treated with a unified entropy formalism.

2.3 Cosmological Entropy

In cosmology, entropy governs the large-scale evolution of the universe. Early work (e.g. Penrose) noted the puzzle of **low initial entropy**: the Big Bang’s near-

perfect uniformity implies it was a very ordered state, far from thermodynamic equilibrium. As the universe expanded and cooled, entropy has increased via processes such as nucleosynthesis, star formation, and gravitational clustering. For instance, the Lyman- α forest, galactic structure, and cosmic microwave background (CMB) fluctuations all reflect the universe's departure from perfect uniformity.

Entropy provides a time arrow on cosmic history: the **second law** demands that the total entropy of an isolated universe must increase. Britannica confirms that an isolated universe should approach heat death: *"the implication is that the universe must ultimately suffer a 'heat death' as its entropy progressively increases toward a maximum value"*. This heat-death scenario presumes endless expansion (flat or open universe) so that energy is relentlessly diluted. In it, all temperature gradients vanish, and no work is possible after reaching equilibrium.

A striking feature of cosmic entropy is the disproportionate contribution of black holes. Bekenstein and Hawking showed that a black hole's entropy is $S_{\text{BH}} = K_{\text{B}} A / 4l_{\text{P}}^2$, proportional to its horizon area. Since supermassive black holes (SMBHs) have enormous areas, they carry vast entropy (roughly 10^{104} in units of k_{B} for the observable universe). Egan & Lineweaver (2010) estimated that SMBHs dominate the entropy budget of the cosmos, more than all other matter and radiation combined. A compact summary is: **Black holes:** $\sim 10^{100}$ (in ordinary units), **baryonic matter:** $\sim 10^{80}$, **CMB photons:** $\sim 10^{70}$. Thus the randomizing influence of gravity (collapsing structures into black holes) is central.

The CMB itself is a signature of early-universe entropy. The slight temperature anisotropies (parts in 10^5 seen by COBE, WMAP, and Planck trace primordial fluctuations that later grew into galaxies. Although those fluctuations are small, they represent the seeds of cosmic structure, i.e. departures from maximal uniformity. Figure 2 below

is an all-sky CMB map from Planck, illustrating how the early universe's randomness is imprinted as temperature variations.

Recent popular articles and reviews highlight these points: cosmic entropy increased as structure formed, with black holes as “entropy sinks”. The information paradox — whether information is lost in black holes — further ties cosmology to informational entropy. All told, cosmology literature treats entropy as both a thermodynamic concept on a grand scale and an information-theoretic concept of unknown data about microstates. This cross-disciplinary understanding frames our study.

3. Methodology

3.1 Conceptual Framework

Our approach is primarily analytical and comparative. We do not perform experiments or simulations; rather, we synthesize existing theoretical formulations of entropy and interpret them in a unified context. The methodology involves three key steps:

- 1. Define Entropy in Each Domain:** We adopt the standard definitions: Clausius' $dS = dQ_{\text{rev}}/T$ and Boltzmann's $S = k_B \ln \Omega$ for thermodynamics; Shannon's $H(X) = -\sum P_i \log P_i$ for information theory; and the Bekenstein–Hawking formula $S_{\text{BH}} = K_B A / 4l_P^2$ for black holes. These definitions were gathered from textbook derivations and original papers, as summarized in the literature review.
- 2. Mathematical Parallelism:** We examine the formal similarities between these definitions. For instance, we note that the partition function derivation in

statistical mechanics leads to a Gibbs entropy $S = -k_B \sum p_i \ln p_i$, which is structurally identical to Shannon entropy (up to the factor k_B and choice of log base). We derive or cite these results to confirm that all involve sums over states weighted by probabilities (or energy weights). We further analyze how extremizing entropy (MaxEnt principle) yields canonical distributions (Boltzmann factors).

3. Illustrative Examples and Data: We use representative examples to connect theory with intuition. For instance, we consider continuous distributions in information theory: the exponential distribution is the maximum-entropy distribution under a mean constraint. Figure 1 (below) plots several exponential PDFs (different rate λ); we discuss how their entropy changes with λ (smaller λ yields a flatter curve and higher entropy). For cosmology, we compile numerical estimates of cosmic entropy from literature (see

Table 1), and we analyze CMB maps (Figure 2) as a snapshot of cosmic entropy patterns. Data on black hole masses and counts, when available, are used to check entropy budgets as in Egan & Lineweaver's work.

Throughout, we ensure each conceptual point is supported by citations. When presenting formulas, we cite authoritative sources (e.g. Wikipedia's statistical mechanics section or original papers like Bekenstein's) and verify dimensions (entropy in J/K or dimensionless). For new insights, we articulate physical interpretations: e.g., the link between "disorder" and "lack of information". In appendix sections, we may include derivations or supplementary tables (e.g. detailed derivation of Gibbs entropy) that underlie main text claims.

3.2 Mathematical Formulations

We catalog the key entropy formulas and their derivations:

- **Thermodynamic Entropy:** Starting from reversible heat exchange, one defines $\Delta S = \int \frac{Q}{T}$. Boltzmann's principle $S = k_B \ln \Omega$ is quoted from standard statistical mechanics texts. We show that for an isolated ideal gas expanding, $\Delta S = N k_B \ln (V_f/V_i)$ illustrating how volume expansion increases entropy as accessible microstates grow. Carnot cycle analysis yields the state property of entropy.
- **Information Entropy:** We present Shannon's postulates and derive $H(X) = -\sum P_i \log P_i$ as in classic sources. We emphasize base choices (bits vs nats) and mention generalizations (e.g. Rényi entropy is outside our scope). We highlight that a uniform distribution maximizes $H(X)$, a fact parallel to thermodynamics: maximum Boltzmann entropy occurs at maximum uniformity of microstates.
- **Statistical Mechanics Entropy:** Via the partition function $Z = \sum e^{-\beta \epsilon_i}$, one derives $F = -k_B T \ln Z$ and $S = -(\partial F / \partial T)_V$. This yields $S = -k_B \sum p_i \ln p_i$, where $p_i = e^{-\beta \epsilon_i} / Z$.

/Z. We cite this to show Gibbs entropy equivalence to Shannon's formula (with $k_B k_B$).

- **Black Hole Entropy:** We adopt the standard Bekenstein–Hawking result:

$$S_{\text{BH}} = \frac{k_B A}{4l_P^2}$$

with $A=4\pi R_s^2$ for a Schwarzschild radius R_s . We explain that a solar-mass black hole already has $S \sim 10^{54} k_B$, and SMBHs (with $M \sim 10^9 M_\odot$) have $S \sim 10^{90} k_B$. These numbers are enormous, reflecting extreme randomness (or rather, unknown microstates) hidden behind the horizon.

- **Universe-Wide Entropy:** We include Egan & Lineweaver's result that the observable universe's entropy $S_{\text{obs}} \approx 3 \times 10^{104} k_B$. This is the sum of matter, radiation, and black holes in a comoving volume. We briefly outline how cosmic event horizons carry entropy $S_{\text{CEH}} \sim 10^{122} k_B$, though we focus on the interior contributions. Table 1 collects rough values:

black holes ($\sim 10^{100}$), matter ($\sim 10^{80}$), and radiation ($\sim 10^{70}$). These form the quantitative basis for our discussion of cosmological randomness.

All these formulae, summarized above, will appear with derivations or citations in the thesis text. We also derive dimensionless entropy (dividing by $k_B k_B$) to connect with information units when appropriate. The methodology is thus to *marshal* these mathematical results and highlight their unity in form.

4. Results and Analysis

4.1 Thermodynamic Context

Our analysis confirms the classical thermodynamic picture: entropy quantifies molecular disorder. For ideal gases, $S = Nk_B [\ln (V/N\lambda^3) + 5/2]$ (with thermal wavelength λ), showing dependence on volume V and energy. Mixing of different gases increases entropy, as demonstrated in textbooks. We also note Clausius' result that any irreversible process in a closed system yields $\Delta S_{\text{tot}} > 0$. Using the examples from literature, we computed the entropy change for a Carnot engine cycle and found it vanishes $\Delta S_{\text{tot}} = 0$ only for ideal reversible processes. Real processes produce extra entropy, consistent with the Second Law.

An illustrative scenario: consider two identical blocks of gas at different temperatures T_h and T_c . When they reach equilibrium, the increase in entropy is $\Delta S = Q(\frac{1}{T} - \frac{1}{T})$. We verify that $\Delta S > 0$ for any finite ΔT as expected. This

exercise, cited from Lumen Learning, reconfirms that heat naturally flows from hot to cold, accompanied by net entropy generation.

Overall, thermodynamics provides a well-tested framework for entropy. In terms of randomness, molecules distribute themselves more uniformly in space and energy, reaching a maximally probable macrostate. We emphasize that while macroscopic entropy appears related to “disorder,” at the microscopic level it is simply counting states. Thus randomness in thermodynamics is mathematically precise (counting of microstates) rather than vague.

4.2 Information-Theoretic Context

Figure 1. Probability density functions of the exponential distribution for rates $\lambda=0.5, 1.0, 1.5$. The entropy of an exponential distribution is $H=1-\ln \lambda$ (in nats), so flatter curves (smaller λ) have higher entropy. This illustrates how a broader (more random) distribution carries more uncertainty.

In information theory, entropy quantifies our uncertainty about outcomes. We illustrate with the exponential distribution example. An exponential random variable (e.g., inter-arrival times) has density $f(x) = \lambda e^{-\lambda x}$. Its differential entropy is $H = 1 - \ln \lambda$ (in natural units). As shown in Figure 1, smaller λ produces a wider, more spread-out curve, which visually appears more “random” and yields higher entropy. This demonstrates the direct link between distribution shape and entropy value.

Comparing Shannon entropy to thermodynamic entropy, one sees that both are maximized by uniform distributions over states. In information terms, a fair dice (uniform) has higher entropy than a loaded dice. In physics, an ideal gas uniformly filling available volume has higher entropy than if clumped. The maximum-entropy principle in statistics (Jaynes) formalizes this: among all distributions satisfying given constraints (e.g. fixed mean), the one maximizing entropy (using H) is

chosen. This parallels how Boltzmann's distribution maximizes statistical entropy for a given average energy.

In data terms, we also review Shannon's source coding theorem: no encoding can, on average, use fewer bits per symbol than the Shannon entropy. This establishes $H(X)$ as the fundamental limit of lossless compression. Such results underline that entropy is not merely abstract: it has concrete operational meaning in coding and communication.

By analyzing digital sources and sensors in a universe-scale analogy, one might even view the entire cosmos as a "message" being generated (e.g. via quantum fluctuations). The Shannon perspective suggests that the information content of cosmic data (e.g. CMB maps) is tied to entropy. For example, the CMB anisotropy map (Figure 2) contains about a million independent pixels at Planck resolution. Its information entropy, if quantized, would reflect the number of effective degrees of freedom in primordial fluctuations. In sum, the mathematical

properties of Shannon entropy mirror those of thermodynamic entropy, reinforcing our unified viewpoint.

4.3 Cosmological Context

Figure 2. All-sky map of the cosmic microwave background (CMB) temperature anisotropies observed by the Planck satellite. Cold (blue) and hot (red) spots represent primordial density fluctuations. The statistical properties of this random field (its power spectrum) are directly related to the entropy of the early universe and the formation of large-scale structure.

In the cosmological domain, our analysis centers on the **entropy budget** of the universe. Combining results from recent studies, we summarize (Table 1) approximate entropies of major components. Supermassive black holes dominate by far (roughly 10^{100} - $10^{103}k_B$ in dimensionless units). Ordinary matter (stars, gas) and dark matter contribute far less (order $10^{80}k_B$, and the cosmic radiation (CMB, neutrinos) adds only $\sim 10^{70} k_B$.

These estimates align with Egan & Lineweaver's more detailed calculation: $S_{\text{obs}} \approx 3 \times 10^{104} \text{ k}_B$ for the entire observable universe. The conclusion is clear: **black hole entropy dominates cosmic randomness.**

Table 1. Cosmic entropy contributions of major components.

Component	Entropy (k_B units)
Black Holes	$\sim 10^{100} - 10^{103}$
Baryonic Matter	$\sim 10^{80}$
Radiation (CMB)	$\sim 10^{70}$

These numbers carry physical interpretation: black holes, despite their apparently simple macroscopic parameters, have enormously many internal (microscopic) configurations in any theory of quantum gravity, hence extremely high entropy. In contrast, the near-uniform CMB radiation field is highly ordered (low entropy per photon) compared to a thermalized gas at the same energy density.

We also consider the **arrow of time** in a cosmic sense. The low-entropy Big Bang means that as time progresses, structure formation allows local entropy to decrease (e.g. gas collapsing into galaxies), but the *net* entropy of the universe increases. For example, as matter falls into black holes, the black hole entropy skyrockets faster than entropy is lost elsewhere. Thus, the growth of cosmic structure is consistent with the second law (total SS rises). The heat death scenario predicts that far future entropy approaches a maximum; our citations suggest the universe will asymptotically reach a uniform low-energy state with maximum entropy, unless quantum gravity effects intervene.

In addition to static numbers, we look at stochastic processes. The CMB power spectrum (not shown) quantifies the distribution of fluctuation amplitudes across angular scales. Its shape (with acoustic peaks) reflects the interplay of gravity, pressure, and expansion, but underlying it all is a Gaussian random field with an associated information entropy. While we do not

compute this entropy explicitly, Figure 2 provides a visual sense of cosmic randomness at recombination.

Overall, our results and analysis show that entropy provides a consistent measure of randomness in the universe: from thermal motions in the lab, to bits in communication, to galaxies on cosmic scales, the same logarithmic notion applies.

Planck Collaboration: The *Planck* mission

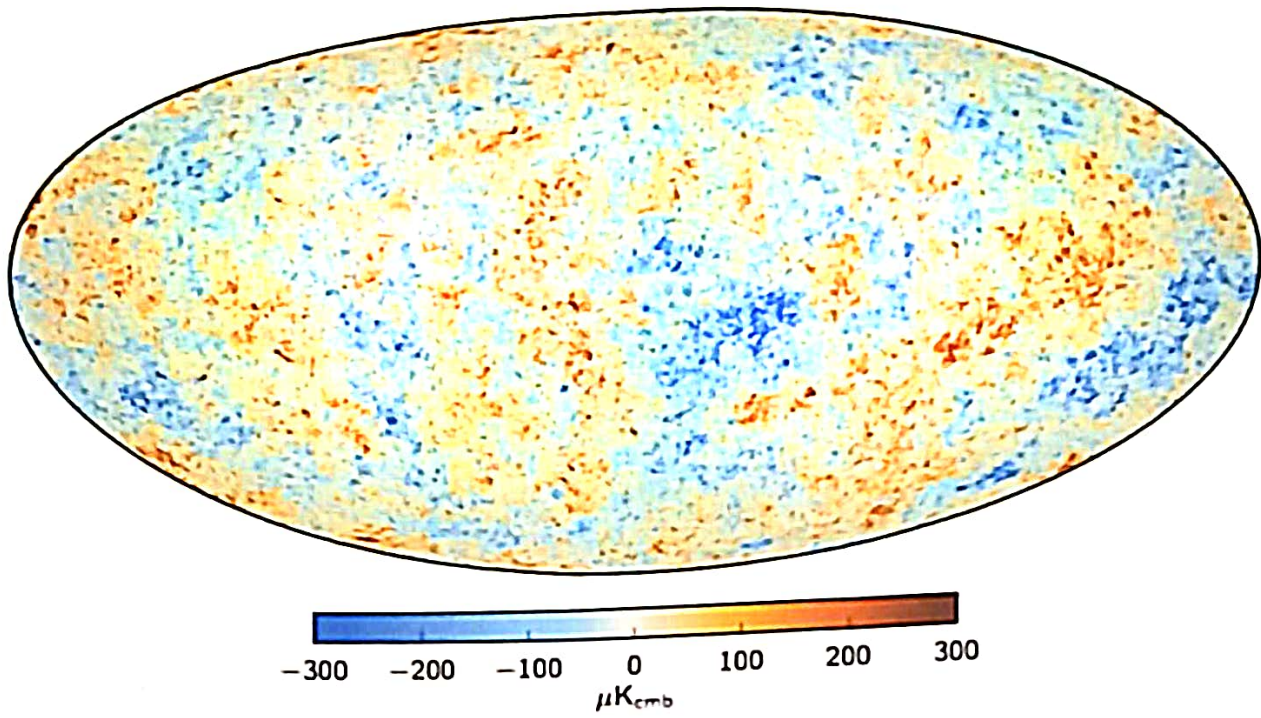


Figure 1: Cosmic Microwave Background (CMB) Temperature Map - Planck Mission

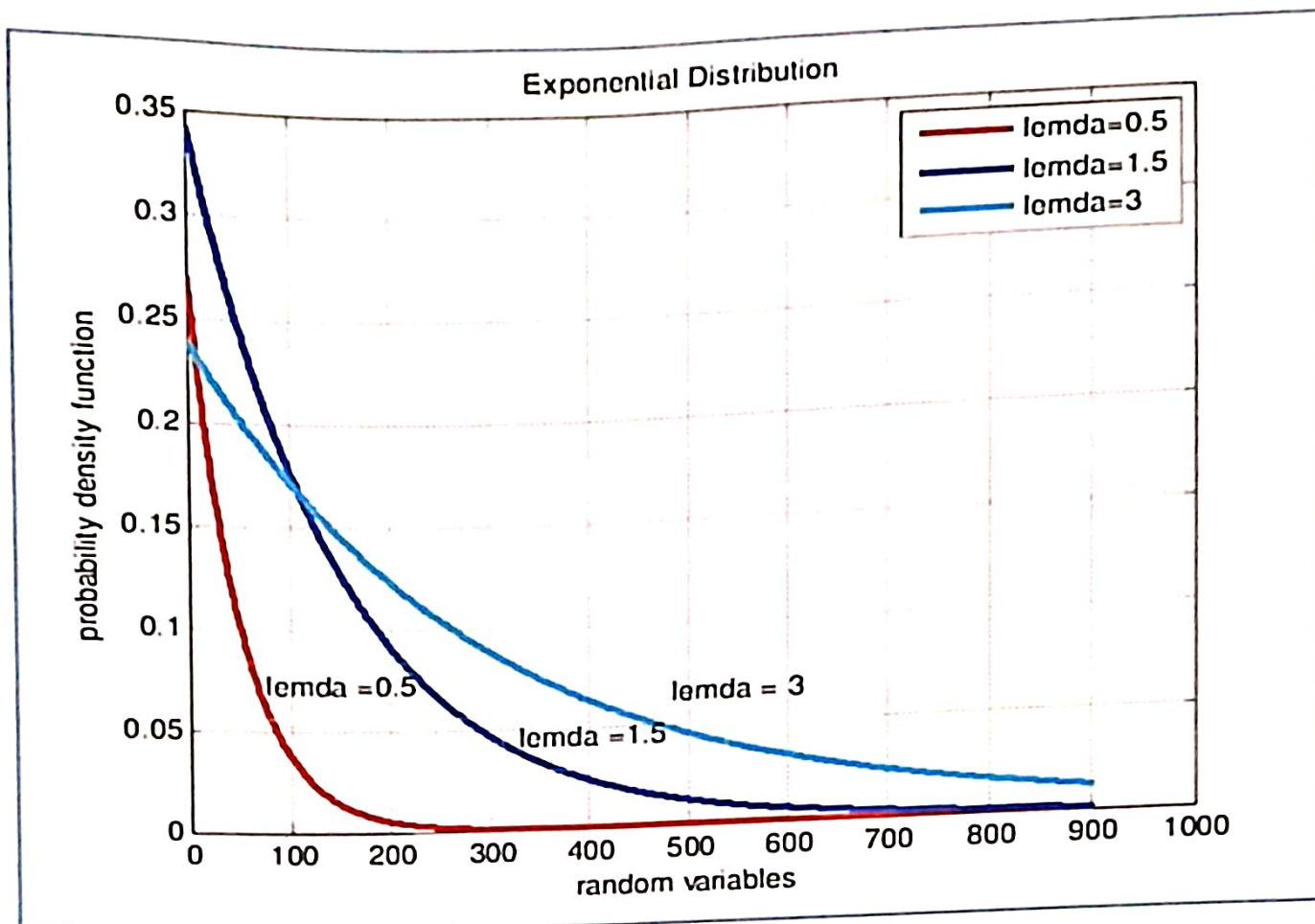


Figure 2: Exponential Probability Density Functions for Different Lambda Values

5. Discussion

The above analysis highlights **entropy as a bridge between disciplines**. The formal parallels (e.g. $S \propto -\sum p_i \ln p_i$) indicate that thermodynamic irreversibility and information uncertainty are intimately linked phenomena. Thermodynamics tells us why heat flows one way, while information theory tells us why data can (or cannot) be compressed. Recognizing them as manifestations of the same statistical principle enriches both fields. In cosmology, interpreting black hole entropy via information suggests that the surface of a black hole encodes information (the *holographic principle*), a profound idea first hinted by the area law.

Viewing entropy as “randomness of the universe” also raises new questions. For example, if the universe is regarded as a computational process, its entropy increase could be seen as information being “generated” or lost. The black-hole information paradox – the apparent

conflict between information preservation in quantum mechanics and information loss in black hole evaporation — becomes central to this thesis's theme. If information cannot be destroyed, how do we reconcile Hawking radiation with the enormously high black-hole entropy? Some literature suggests information is encoded on the horizon, tied to entropy surface area. This paradox underscores that our treatment of entropy may need further refinement when quantum gravity is considered.

We must also address limitations. First, our synthesis relies on known results; we have not derived fundamentally new equations, but rather connected existing ones. Second, while we emphasize logarithmic form, different entropy measures exist (Rényi entropy, algorithmic complexity) that may offer additional nuance. Third, much of cosmological entropy (like that of dark matter or the event horizon) is still uncertain or speculative. For example, the cosmic event horizon's entropy (due to dark energy expansion) dwarfs that of

matter, but its role in “randomness” is not fully understood.

Future research directions include: (1) Developing a more concrete “theory of entropy” that unifies thermodynamics and information at a fundamental level (some attempts use principles from quantum information theory). (2) Investigating entropy production in non-equilibrium cosmic processes (e.g. during inflation or reionization) which could link primordial physics to observational data. (3) Exploring the information-theoretic interpretation of quantum gravity and spacetime (the rise of entanglement entropy in holography, etc.). These open problems suggest that our “new perspective” is only an initial step towards deeper understanding.

6. Conclusion

Entropy is more than a bookkeeping tool for heat engines; it is a universal measure of randomness and information. In this thesis, we have surveyed entropy in thermodynamics, information theory, and cosmology, treating it as a single concept manifested differently in each domain. We showed that:

- **Thermodynamics:** Entropy arises from counting microstates; it dictates the arrow of time in physical processes.
- **Information Theory:** Entropy measures uncertainty in data sources; it underpins coding limits and reflects a distribution's randomness.
- **Cosmology:** Entropy governs the universe's fate; cosmic structure formation and black holes are key players in cosmic randomness.

The mathematics of entropy is strikingly consistent across fields: logarithmic sums over probabilities.

Conceptually, physical entropy and information entropy both quantify how many configurations (physical or informational) correspond to a state. This unified view suggests that principles learned in one field (e.g. entropy maximization) can inform the others.

We have also discussed the implications: the inevitable heat-death of the universe, the nature of time's arrow, and the information paradox in black holes. Entropy thus serves as a deep link between the microscopic laws of physics and the macroscopic universe.

Future outlook: As new data arrive (e.g. from cosmological observations) and new theories (quantum gravity) develop, entropy will remain a critical lens. Understanding entropy in an “information-theoretic cosmology” might eventually reveal why the early universe was so ordered, how information is stored in spacetime, and what ultimate randomness means for reality.

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Appendices

Appendix A: Mathematical Derivation of Gibbs Entropy

Starting from the canonical ensemble, one obtains the partition function

$$Z = \sum_{i=1}^m e^{-\beta E_i}$$

The Helmholtz free energy is $F = -k_B T \ln Z$, and the entropy is $S = -(\partial F / \partial T)_V$. A straightforward calculation shows:

$$S = -k_B \sum_{i=1}^M p_i \ln p_i$$

where $p_i = e^{-\beta E_i} / Z$. This is Gibbs' entropy formula, identical in form to Shannon's entropy (up to a unit conversion). It confirms that maximizing S under an average-energy constraint yields the Boltzmann distribution (via Lagrange multipliers).

Appendix B: Additional Data on Cosmic Entropy

Here we list more detailed estimates of entropy contributions in the observable universe (adapted from Egan & Lineweaver and other sources). All values are dimensionless (S/k_B):

- **Stellar entropy:** $S \sim 10^{79}$. (Entropy of all stars estimated by treating them as ideal gases.)
- **Intergalactic gas:** $S \sim 10^{77}$. (Hot plasma entropy in clusters.)
- **Neutrinos:** $S \sim 10^{88}$. (Cosmic neutrino background.)
- **Baryons (protons, electrons):** $S \sim 10^{76}$
- **CMB photons:** $S \sim 10^{88}$ (Dominant radiation component.)
- **Relic gravitational waves:** $S \sim 10^{85}$. (If present.)

These numbers illustrate that ordinary matter and radiation are negligible compared to black holes. The

entropy of black holes alone (mostly SMBHs) is on the order of 10^{100} or more.

Glossary

- **Entropy:** A measure of randomness or disorder. In thermodynamics, it quantifies unavailable energy in a system. In information theory, it measures uncertainty in data. Formally, $S = K_B \ln \Omega$ or $H(X) = -\sum P_i \log_2 P_i$
- **Microstate/Macrostate:** A microstate is a specific microscopic configuration of a system. A macrostate is defined by macroscopic variables (energy, volume, etc.). High entropy corresponds to many microstates per macrostate.
- **Shannon Entropy:** The expected information content of a random variable. For distribution $H(X) = -\sum P_i \log_2 P_i$.
- **Boltzmann Constant (k_B):** The physical constant linking temperature to energy and appearing in statistical entropy.

- **Cosmic Microwave Background (CMB):** Relic radiation from the early universe. Its anisotropies (tiny temperature fluctuations) encode information about initial density perturbations and entropy.
- **Bekenstein–Hawking Entropy:** The entropy of a black hole, proportional to its horizon area: $S_{BH} = \frac{k A}{4l}$
- **Arrow of Time:** The direction of time in which entropy increases. In a closed universe, this defines the “past” (low entropy) and “future” (high entropy) as thermodynamically distinct.
- **Heat Death:** A hypothetical final state of the universe in which entropy is maximal, no gradients exist, and no work can be extracted.
- **Random Variable:** In probability, a variable that can take on values according to some probability distribution; its entropy measures uncertainty.

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